



**Emily Riehl**

Johns Hopkins University

# A conversation on professional norms in mathematics

Hardy Lecture Series, Bristol

## Inspiration: the Liberated Mathematician



*Respected research math is dominated by men of a certain attitude. Even allowing for individual variation, there is still a tendency towards an oppressive atmosphere, which is carefully maintained and even championed by those who find it conducive to success. As any good grad student would do, I tried to fit in, mathematically. I absorbed the atmosphere and took attitudes to heart. I was miserable, and on the verge of failure. The problem was not individuals, but a system of self-preservation that, from the outside, feels like a long string of betrayals, some big, some small, perpetrated by your only support system. When I physically removed myself from the situation, I did not know where I was or what to do. First thought: FREEDOM!!!! Second thought: but what about the others like me, who don't do math the "right way" but could still greatly contribute to the community? I combined those two thoughts and started from zero on my thesis.*



— Piper H.,

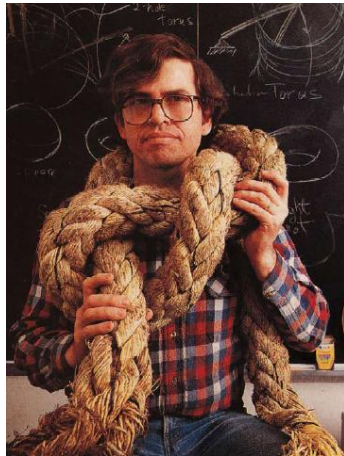
“The Equidistribution of Lattice Shapes of Rings of Integers of Cubic, Quartic, and Quintic NumberFields: an Artist's Rendering Based on the original story by Manjul Bhargava and Piper Harron”

# Inspiration: on Proof and Progress in Mathematics



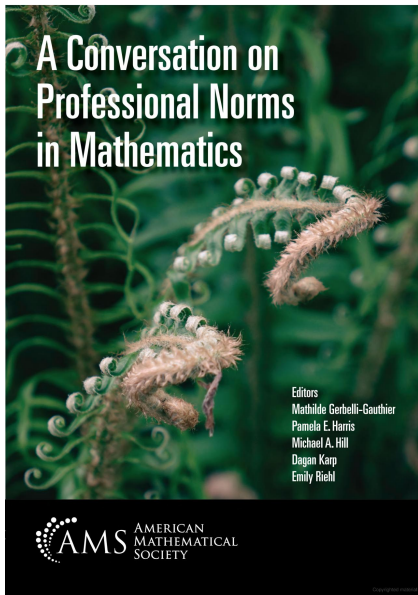
*We mathematicians need to put far greater effort into communicating mathematical ideas. To accomplish this, we need to pay much more attention to communicating not just our definitions, theorems, and proofs, but also our ways of thinking. We need to appreciate the value of different ways of thinking about the same mathematical structure.*

*We need to focus far more energy on understanding and explaining the basic mental infrastructure of mathematics—with consequently less energy on the most recent results. This entails developing mathematical language that is effective for the radical purpose of conveying ideas to people who don't already know them.*



— Bill Thurston,  
“On proof and progress in mathematics”

# A conversation on professional norms in mathematics



## A conversation on professional norms in mathematics

[Johns Hopkins University](#)

September 20-22, 2019

Sponsored by the National Science Foundation DMS-1652600 and the [Department of Mathematics](#) at Johns Hopkins University.

Contact [Emily Riehl](#) ([@emilyriehl](#)) with questions.



# Workshop presentations



- Eugenia Cheng, "Inclusion-exclusion in mathematics and beyond: who stays in, who falls out, why it happens, and what we could do about it"
- Alexander Diaz-Lopez, "Becoming a better version of ourselves"
- Pamela E. Harris, "Avoiding the academic savior complex: How to mentor underrepresented faculty"
- Denis R. Hirschfeldt, "Mathematicians, Collective Bargaining, and the Corporatized Academy"
- Mike Hill, "Queer spheres"
- Dagan Karp, "An Introduction to Critical Theory in Postsecondary Mathematics Education"
- Oliver Knill, "On parameters for communicating mathematical ideas"
- David Kung, "From Teaching Math to Teaching Students: Transforming classroom norms among college math instructors"
- Izabella Łaba, "Rethinking universities in the era of climate change"
- Luis A. Leyva, "Racialized and Gendered Mechanisms of Pre-calculus and Calculus Instruction: A Window into Cultural and Professional Norms in Mathematics"
- Michelle Manes, "Be the change you want to see in mathematics"
- Adriana Salerno, "The mathematician as public intellectual"
- Francis Su, "The College/University Divide: How Do We Fix It?"
- Aris Winger, "Acknowledgement of the Impact Race has in our Practice"



- William Yslas Vélez and Ana Christina Vélez, “The Time for Miracles is Over”
- Pamela E. Harris, “On Toxic Mentorship and the Academic Savior Complex”
- Federico Ardila–Mantilla, “Todxs Cuentan: Building Community and Welcoming Humanity from the First Day of Class”
- Eugenia Cheng, “Congressional Question Time”
- Michelle Manes, “Mathematics, We Have a Problem”
- Dagan Karp, “Fiber Bundles and Intersectional Feminism”
- Oliver Knill, “On Parameters for Communicating Mathematics”
- Denis R. Hirschfeldt, “Turning Coffee into Unions: Mathematicians and Collective Bargaining”
- Izabella Łaba, “Universities in the Time of Climate Change”



1. Todxs cuentan: building community and welcoming humanity from the first day of class (Federico Ardila–Mantilla)
2. Turning Coffee into Unions: Mathematicians and Collective Bargaining (Denis R. Hirschfeldt)
3. Universities in the time of climate change (Izabella Łaba)
4. A norm for machine-generated mathematical proof (personal thoughts)



1

Todxs cuentan: building community and  
welcoming humanity from the first day of class  
(Federico Ardila–Mantilla)



# Welcoming humanity from the first day of class



Those of us who aspire to teach and practice mathematics for human flourishing are confronted with a challenge of how to foster a welcoming community in the classroom.

**Federico Ardila–Mantilla** opens with an exercise in which he and his students describe something outside of mathematics that they love doing and share how they feel about being in the class because:



*I wanted us to say these things out loud to remember that mathematics is a human endeavor. I hoped to make clear that our full humanity was not only welcome here, but in fact would define and enrich our mathematical space.*

# Community agreement



He and his students then collaborate on a community agreement:



math 420/720 . combinatorics

spring 2017

san francisco state university

**Community Agreement.** This course aims to offer a joyful, meaningful, and empowering experience to **every** participant; we will build that rich experience together by devoting our strongest available effort to the class. You will be challenged and supported. Please be prepared to take an active, critical, patient, and generous role in your own learning and that of your classmates.

This is adapted iteratively from semester to semester  
in collaboration with the students from each new course.

## Making space, taking space, holding space



Ardila–Mantilla emphasizes that students' sense of belonging must be nourished constantly throughout the semester. Part of this is achieved by a collaboratively developed community agreement on classroom culture and atmosphere that emphasizes the practices of **making space** and **taking space**:

*If I feel comfortable speaking out, I should be mindful of how much space I take, and make room for others. If I tend to be quieter in groups, I should remember that my ideas are important, and others will benefit from hearing them.*

Ardila–Mantilla models these practices in his essay, in which he also demonstrates how to **hold space**, meaning “supporting someone by being fully present for them to process their feelings, without letting one’s own feelings, ego, or proposed solutions interfere with that process” by including his students’ own experiences in his classroom, told in their own words.

## Making space, taking space, holding space



*As a mathematics researcher with more than 20 years of experience, I feel pretty confident that my mathematical ideas are valuable. It sometimes takes a special effort to truly listen to students' ideas without projecting my own views onto them. When I have been able to really make space for students' thought, we have all learned very innovative and useful ways of thinking about combinatorics.*

*As a mathematics educator with great interest but under 20 minutes of formal training in education, I still feel like a student with everything to learn. Writing about pedagogy feels very uncomfortable. For every criticism the reader may have of my educational work, I have at least five. I cannot count the number of self- deprecating statements I have edited out of this note.*

# Axioms for mathematical education and outreach



In an earlier version of *Todos Cuentan*, *Ardila–Mantilla* propose axioms “that have been a useful foundation for my educational and outreach efforts. I also find them far from perfect, a constant work in progress, as any work of this sort must be.”

- Axiom 1.** Mathematical potential is equally present in different groups, irrespective of geographic, demographic, and economic boundaries.
- Axiom 2.** Everyone can have joyful, meaningful, and empowering mathematical experiences.
- Axiom 3.** Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.
- Axiom 4.** Every student deserves to be treated with dignity and respect.

*“These statements should not sound revolutionary, and considering the current practices of the mathematical society, they are a pressing call to action.”*



2

## Turning Coffee into Unions: Mathematicians and Collective Bargaining (Denis R. Hirschfeldt)

# Solidarity across the academy



Denis R. Hirschfeldt makes an urgent call for a fully unionized academy of

“faculty, student workers, other academic staff,  
and of course nonacademic staff who are as much  
a part of the endeavor as we are,”

noting this imperative is particularly challenging for  
certain mathematics faculty because

“the concept of solidarity has to be learned by those  
with the luxury never to have had to rely on it.”

## Faculty alignment with management?



In a 1980 Yeshiva University case, the United States Supreme Court held that

*“[t]he University’s full-time faculty members are managerial employees excluded from the [National Labor Relations] Act’s coverage.”*

The dissent argues:

*Unlike industrial supervisors and managers, university professors are not hired to “make operative” the policies and decisions of their employer. Nor are they retained on the condition that their interests will correspond to those of the university administration. Indeed, the notion that a faculty member’s professional competence could depend on his undivided loyalty to management is antithetical to the whole concept of academic freedom. Faculty members are judged by their employer on the quality of their teaching and scholarship, not on the compatibility of their advice with administration policy.*



## Privilege, gratitude, and responsibility



Hirschfeldt acknowledges the gratitude that many of us feel that we have jobs that pay us “to do something we would want to do anyway” but argues that we should support academic and nonacademic unions because, not despite, how fortunate we feel:

*To the extent that we feel privileged to do what we do, it is essential to understand that our work depends on a network of labor to which we do a great disservice if we stand apart from it. ...So any sense of gratitude we might have ought to be expressed by a commitment to stand in solidarity with those who make it possible directly, as well as those who have been hurt and disadvantaged by this process.*

Hirschfeldt questions the long-term sustainability of the service teaching model, as free-market funding-allocation policies encourages competition between departments, and frames tenure as “increasingly becoming part of an unsustainable divide between academic haves and have-nots that threatens ultimately to swallow even the haves.”

## Lessons from the pandemic



Hirschfeldt's essay, really a seminar in critical university studies, concludes with three lessons of the pandemic, the first being:

*Things are fine until they're not,  $n^2 + n + 41$  is always prime until it isn't, and the cost of not being prepared can be staggering. This is true at the individual level, and all the more so at the communal one. Asking why someone without major grievances would want to unionize is like asking why a healthy person would want to get health insurance. But no, it isn't. It's like asking why a healthy person would want to have a robust public health system.*

Echoing Michelle Manes' critique of sex-based discrimination policies in the academy, a second lesson is that "when things go wrong, current institutional structures will not save us." A third lesson is directed at those who recognize all the drawbacks of collective bargaining:

*To riff on Churchill, collective solutions are the worst solutions to our problems except for all of those others that have been tried.*



3

## Universities in the time of climate change (Izabella Łaba)

## Anticipating a pandemic



Izabella Łaba's prescient essay is based on a presentation on the same topic that, fittingly, she elected to deliver remotely over Zoom to avoid a trip across the continent. Her narrative interweaves apprehensions from the Fall of 2019 which "grew out of my frustration with common institutional responses to the climate emergency" with insights that take on even greater urgency in light of the global pandemic which started a few months later.



While administrators write "fanfics about their endless emails, memos and directives providing support and leadership during that difficult time" she warns that "this is only a small preview of what climate change will bring" and that moving all classes online for a year or two is far from the worst that can happen.

# The pandemic as a preview



Łaba continues:

*Yet, this is only a small preview of what climate change will bring. Other pandemics may follow, as we continue to encroach on parts of the biosphere that would be best left to themselves. As wildfires ravaged Australia a few months ago, we watched helplessly the cell phone videos from residents who, having been told that it was too late to leave, sought shelter by wading into the ocean instead. Extreme conditions will become commonplace. There will be no help coming and no one available to bail us out when everyone's resources are strained to the limit.*

Universities have not acquitted themselves well with their response:

*We could continue in this manner, asking people to perform heroic feats so that institutions could pretend that everything is fine.*

*Or we could acknowledge the reality.*

*We could slow down, scale back, allow room for buffer in case of emergencies or unexpected circumstances.*

## A change in personal habits



In response to calls to “ditch our cars, bike to work, avoid plastic bags and disposable food containers, sort our recyclables, turn down the thermostat and put on a sweater”:

*Faculty often report 50-60 hour work weeks with little or no vacations. Class sizes are increasing. Digitization, instead of reducing our administrative workload, has increased it by redirecting much of the work from staff to faculty as self-service. Tenured faculty already do research, supervise graduate students, write grant proposals, serve as journal editors and referees. We are also asked to learn innovative teaching methods, monitor and support student wellbeing, engage in public outreach, and participate in initiatives to promote diversity and inclusion. These are good things to do, but can one person really do it all? In the limited time we have?*

Łaba notes “Tired and overworked people do not have the time or capacity to accept additional challenges. They will drive to work, order takeout food for lunch or dinner even if it comes in Styrofoam containers, forget their reusable bags, throw garbage in the compost bin by mistake, generally waste resources that could otherwise be saved.”

## A Green New Deal for Universities



Instead, [Laba](#) calls for a “Green New Deal” for universities, with a redistribution of both salaries and workload, creating new jobs while allowing faculty “more time and capacity to have a life outside of work”:

*Faculty numbers, especially the numbers of tenured and tenure track faculty, have no relation to how much work actually needs to be done at universities. Our workloads have long been ballooning out of control. New responsibilities are added almost every day. At the same time, faculty positions continue to be eliminated or converted to temporary ones, so that the increasing total workload is shared between fewer faculty.*

She also calls for [less gatekeeping and more redistribution](#):

*“We spend an astounding amount of time and energy on gatekeeping: refereeing, proposal evaluations, ranking decisions, writing and reading recommendation letters, editorial work. What if we did not have to do that?  
Or, at least, if we could reduce it by half or more?”*

## Slower habits of work



Łaba contends that slower habits of work may even improve its quality. With more equality, less competition, and more cooperation mathematicians could “spend less time ‘producing’ new papers making incremental progress, and pay more attention to consolidation, exposition and preservation of the knowledge we already have.”

*Change will be forced on us. We will have to adapt, one way or another. It's up to us whether we make the transition humane and how much of human knowledge we manage to preserve. We cannot buy our way out of the climate emergency. Capitalism will not save us. Universities, as non-profit organizations dedicated to the pursuit and dissemination of knowledge, should be leading the way. We should experiment and then model the change for others. We will need to learn to make do with less. We like to say that mathematics only requires a pen and pencil. We may be tested on that.*





4

A norm for machine-generated mathematical  
proof (personal thoughts)

# Vibe proving



Is it true that an  $\infty$ -category is idempotent complete (in the  $\infty$ -categorical sense) if and only if its homotopy category is idempotent complete (in the 1-categorical sense)?



Show thinking



Yes, it is true that an  $\infty$ -category  $\mathcal{C}$  is idempotent complete (in the  $\infty$ -categorical sense) if and only if its homotopy category  $ho(\mathcal{C})$  is idempotent complete (in the 1-categorical sense).



Let's establish the definitions and then prove the equivalence.

...

## Conclusion:

The two conditions are equivalent. The definitions of idempotent completeness are "homotopy-invariant" in a way that translates directly between the  $\infty$ -category and its homotopy category. This result is also confirmed by the fact that  $h(Kar(\mathcal{C})) \cong Kar(h\mathcal{C})$ , where  $Kar(-)$  denotes the Karoubi envelope (idempotent completion) and  $h(-)$  denotes taking the homotopy category. An  $(\infty)$ -category is idempotent complete if and only if the canonical functor into its Karoubi envelope is an equivalence.

The omitted text contains a clearly written argument with a subtle error, the sort of optimistic mistake often made by human mathematicians. **The conclusion is incorrect.**

Your answer uses the "incoherent" definition of an idempotent in an  $\infty$ -category? Can you answer the same question using the coherent definition of an idempotent in an  $\infty$ -category?



Show thinking



You're right to press on the subtleties of "coherence" in  $\infty$ -categories. Let's clarify how these definitions are typically handled and why the previous conclusion still holds.

...

If a framework requires a more elaborate "coherent idempotent object" (e.g., an object in a category of idempotents, or an algebra for a certain monad), the definition of "idempotent completeness of  $\mathcal{C}$ " itself usually still boils down to the splitting condition for any  $e$  satisfying  $e \circ e \simeq e$ . The power of  $\infty$ -category theory is that such "weak" conditions, when interpreted within a coherently structured environment, often suffice and lead to robust theories.

Thus, the answer remains the same: Yes, an  $\infty$ -category is idempotent complete (in the standard  $\infty$ -categorical sense) if and only if its homotopy category is idempotent complete (in the 1-categorical sense). The "coherence" is sufficiently handled by the definitions used.

# How do we defend the mathematical literature against vibe proving?



A lot of the mathematical text generated by top “reasoning” models is pretty good and all of it looks good.\*

\*Caveat: large language models tend to get even the simplest numerical calculations horribly wrong.

But pretty good is not good enough: one minor error could make an entire logical argument reach the wrong conclusion: **true** instead of **false**.

As a journal editor specializing in a subfield where papers tend to be long, technical, and difficult to referee, I am **extremely concerned** about AI generated mathematical “proofs.”



Why has mathematics largely avoided the replication crisis that has confronted other fields?

**Peer review in theory:** careful refereeing should lead to an error-free publications.

Unfortunately, the mathematical literature contains famous mistakes as well as contradictory theorems.

**Peer review in practice:** in theory any proof should be **reproducible** by any reader — allowing the reader to understand for themselves why the result is true.

When papers have enough readers, mistakes are eventually caught.

**Importantly:** human mathematicians are careful in claiming they have a proof.

# Proof as peer review?



But sometimes these ideals break down:

One Fields medalist was dismayed to find mistakes in his published, well-studied papers:

*“A technical argument by a trusted author, which is hard to check and looks similar to arguments known to be correct, is hardly ever checked in detail.”*

— Vladimir Voevodsky

Another Fields medalist expressed doubts about a particular proof he had discovered — and also doubted that anyone else would check it:

*“...while I was very happy to see many study groups on condensed mathematics throughout the world, to my knowledge all of them have stopped short of this proof. (Yes, this proof is not much fun...)”*

— Peter Scholze

# A new paradigm for proof writing



Computer formalization is a new and not yet widely practiced method of **developing** and **communicating** rigorous mathematical proofs **interactively** through the use of a computer program called a **computer proof assistant**.

- The mathematician inputs each line of their proof in a precise syntax.
- The computer checks that the logical argument supplied by the user produces a valid deduction of the claimed mathematical statement.
- Depending on the sophistication of the computer program, it might also “assist” the mathematician in various ways:
  - By catching errors of reasoning (unjustified assumptions, missing cases, etc).
  - By keeping track of the current state of a complicated logical argument.
  - By suggesting or even automatically generating proofs (autoformalization).

**Aside:** modern proof assistants often use a newer formal system — **dependent type theory** — in place of traditional Zermelo-Fraenkel set theory and first order logic.

# A new paradigm for proof checking

*“A technical argument by a trusted author, which is hard to check and looks similar to arguments known to be correct, is hardly ever checked in detail.”*

— Vladimir Voevodsky

*“...while I was very happy to see many study groups on condensed mathematics throughout the world, to my knowledge all of them have stopped short of this proof. (Yes, this proof is not much fun...)”*

— Peter Scholze

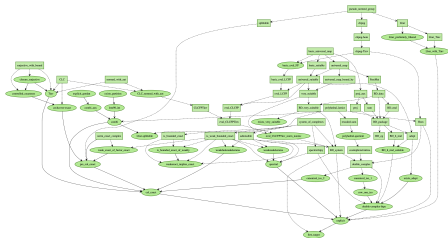
Voevodsky and Scholze both turned to computer formalization to resolve their doubts about the veracity of their own proofs.

MATHEMATICS

## The Origins and Motivations of Univalent Foundations

*A Personal Mission to Develop Computer Proof  
Verification to Avoid Mathematical Mistakes*

By Vladimir Voevodsky • Published 2014



# A new paradigm for mathematical proof?



Is there consensus in the mathematical community about the computer formalized proof paradigm?

**Absolutely not!** This is very much a minority point of view.

- I would estimate that most departments contain **few if any** mathematicians who are actively working with computer proof assistants.
- Even early adopters would agree that computer proof assistants are currently **too hard to use** for most day-to-day proof writing.
- The challenges presented by formalization vary tremendously between subfields, and their solutions may require new ideas — perhaps new domain-specific foundations?

In addition, there are active debates about how such a shift would affect **human understanding of mathematics**, the real point of what we do.



# A norm for machine-generated mathematical proof



Despite well-known imperfections, the mathematical community can take deep pride in our overwhelmingly reliable and continually improving standards for mathematical proof.

We should demand the same for AI when it comes to the mathematical realm.

Maintaining high standards will frustrate near term progress, delaying the arrival of a machine we validate as having “artificial mathematical intelligence,” but should be beneficial for overall reliability in the long run, in mathematics and beyond.

Specifically, I want to propose the following norm for the mathematical community when it comes to original mathematics produced by an AI system:

Any artificially generated mathematical text will **not be considered as a proof** unless:

- It has been communicated in both a natural language text paired with a computer formalization of all definitions, theorems, and proofs.
- The formalization has been accepted by the proof assistant and human expert referees have vetted both the formalization and the paired text.

# A conversation on professional norms in mathematics



## A Conversation on Professional Norms in Mathematics

American Mathematical Society, 2021: 138pp.

Edited by: Mathilde Gerbelli-Gauthier, Pamela E. Harris,  
Michael A. Hill, Dagan Karp, and Emily Riehl.

- [fardila.com/Articles/firstdayofclass.pdf](https://fardila.com/Articles/firstdayofclass.pdf)
- [math.uchicago.edu/~drh/Papers/Papers/unions.pdf](https://math.uchicago.edu/~drh/Papers/Papers/unions.pdf)
- [ilaba.wordpress.com/2020/08/16/universities-in-the-time-of-climate-change/](https://ilaba.wordpress.com/2020/08/16/universities-in-the-time-of-climate-change/)

Thank you!

